Constacyclic Codes over $\mathbb{Z}_p + u\mathbb{Z}_p + ... + u^{k-1}\mathbb{Z}_p$

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Abstract

Let R be the ring $\mathbb{Z}_p + u\mathbb{Z}_p + ... + u^{k-1}\mathbb{Z}_p$, $k > 1$ which is a chain ring of characteristic p and with residue field $\mathbb{Z}_p$. In this article, we study all constacyclic codes of arbitrary length N over R. First, we establish unique polynomial representations for constacyclic codes of length p^s over a Galois extension of R, where $s = v_p(N)$ and v_p is the p-adic valuation. Then we use Discrete Fourier Transform (DFT) to determine constacyclic codes of length N over R. In addition, Hamming distance and dual of such codes are obtained.

Keywords

constacyclic code, repeated-root code, coding over ring, Discrete Fourier Transform, Hamming distance.

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